**Chapter 2**

Table of Contents

[Chapter 2.1 3](#_Toc66760100)

[Sets 3](#_Toc66760101)

[Equal Sets 4](#_Toc66760102)

[Subsets 4](#_Toc66760103)

[Empty Set or Null Set 4](#_Toc66760104)

[Singleton Set 4](#_Toc66760105)

[Venn Diagrams 5](#_Toc66760106)

[Cardinality 5](#_Toc66760107)

[Power Sets 5](#_Toc66760108)

[Ordered Pairs 6](#_Toc66760109)

[Cartesian Products 6](#_Toc66760110)

[Chapter 2.2 7](#_Toc66760111)

[Union of Sets 7](#_Toc66760112)

[Intersection of Sets 7](#_Toc66760113)

[Disjoint Sets 7](#_Toc66760114)

[Difference of Sets 7](#_Toc66760115)

[Complement of Sets 7](#_Toc66760116)

[Set Identities 8](#_Toc66760117)

[Computer Representation of Sets 9](#_Toc66760118)

[Chapter 2.3: Functions 10](#_Toc66760119)

[One-to-One Functions or Injunctions 10](#_Toc66760120)

[Onto Functions or Surjective Functions 10](#_Toc66760121)

[One-to-One Correspondence or Bijections 11](#_Toc66760122)

[Inversion 11](#_Toc66760123)

[Flooring and Ceiling 11](#_Toc66760124)

[Composite Functions 13](#_Toc66760125)

## Chapter 2.1

### Sets

A set is an unordered collection of objects, called elements or members of the set.

- is an element of set

- is not an element of set

- roster method

- set builder method

- set builder method

#### Common Notations

- the set of natural numbers

- the set of integers

- the set of positive integers

- the set of rational numbers

- the set of real numbers

- the set of positive real numbers

- the set of complex numbers

- universal set

### Equal Sets

Two sets are equal if they have the same elements. The order of the elements and the repetition of the same element are irrelevant.

If and then

### Subsets

A set is called a subset of another set if every element of is also an element of .

- is a subset of

- is not a subset of

Every non-empty set () is guaranteed to have at least two subsets, and the set itself .

### Empty Set or Null Set

A special set with no elements. It is denoted by or .

is not an empty set. It is a singleton set with the single member, which is the empty element itself.

### Singleton Set

A set with a single element.

### Venn Diagrams

These are graphical representations of sets. The universal set includes all objects under consideration and is represented by a rectangle. Circles are used to represent different sets inside the rectangle. Points are used to represent particular elements of a set.

### Cardinality

The cardinality of a set is simply the number of distinct elements within that set. For a set that contains odd positive integers less than 10,

### Power Sets

The power set of a set is the set of all subsets of the set . It is denoted by . The cardinality of a power set is , where is the cardinality of the set .

The empty set and the set itself are members of the power set.

For an empty set,

For a set ,

### Ordered Pairs

An ordered -tuple is a collection of elements in order, from to . Two ordered -tuples are considered equal if and only if each corresponding pair of their elements is equal, i.e. for each value of from to .

In particular ordered -tuples are called ordered pairs. Thus, the ordered pairs and are equal if and .

### Cartesian Products

The cartesian product of two sets and , denoted by is the set of all ordered pairs where and .

## Chapter 2.2

### Union of Sets

The union of set and is the set that contains those elements that are present in either or or both. It is denoted by .

### Intersection of Sets

The intersection of set and is the set that contains those elements that are present in both and . It is denoted by .

### Disjoint Sets

Two sets are called disjoint if their intersection is the empty set (no common members).

### Difference of Sets

The difference of two sets and is denoted by . It is the set that contains the elements that are present in but not in .

### Complement of Sets

The complement of a set is simple , where is the universal set. It is denoted by .

### Set Identities

|  |  |
| --- | --- |
|  | Identity Laws |
|  |
|  | Domination Laws |
|  |
|  | Idempotent Laws |
|  |
|  | Complementation Law |
|  | Commutative Laws |
|  |
|  | Associative Laws |
|  |
|  | Distributive Laws |
|  |
|  | De Morgan’s Laws |
|  |
|  | Absorption Laws |
|  |
|  | Complement Laws |
|  |

### Computer Representation of Sets

Let the universal set have an arbitrary ordering of elements to , each of which are represented by a bit. Let the subset contain some of those elements. The bit will be if is a member of , and otherwise.

For odd integers in , the bit string representation will be .

Unions and intersections can also be done with bit strings.

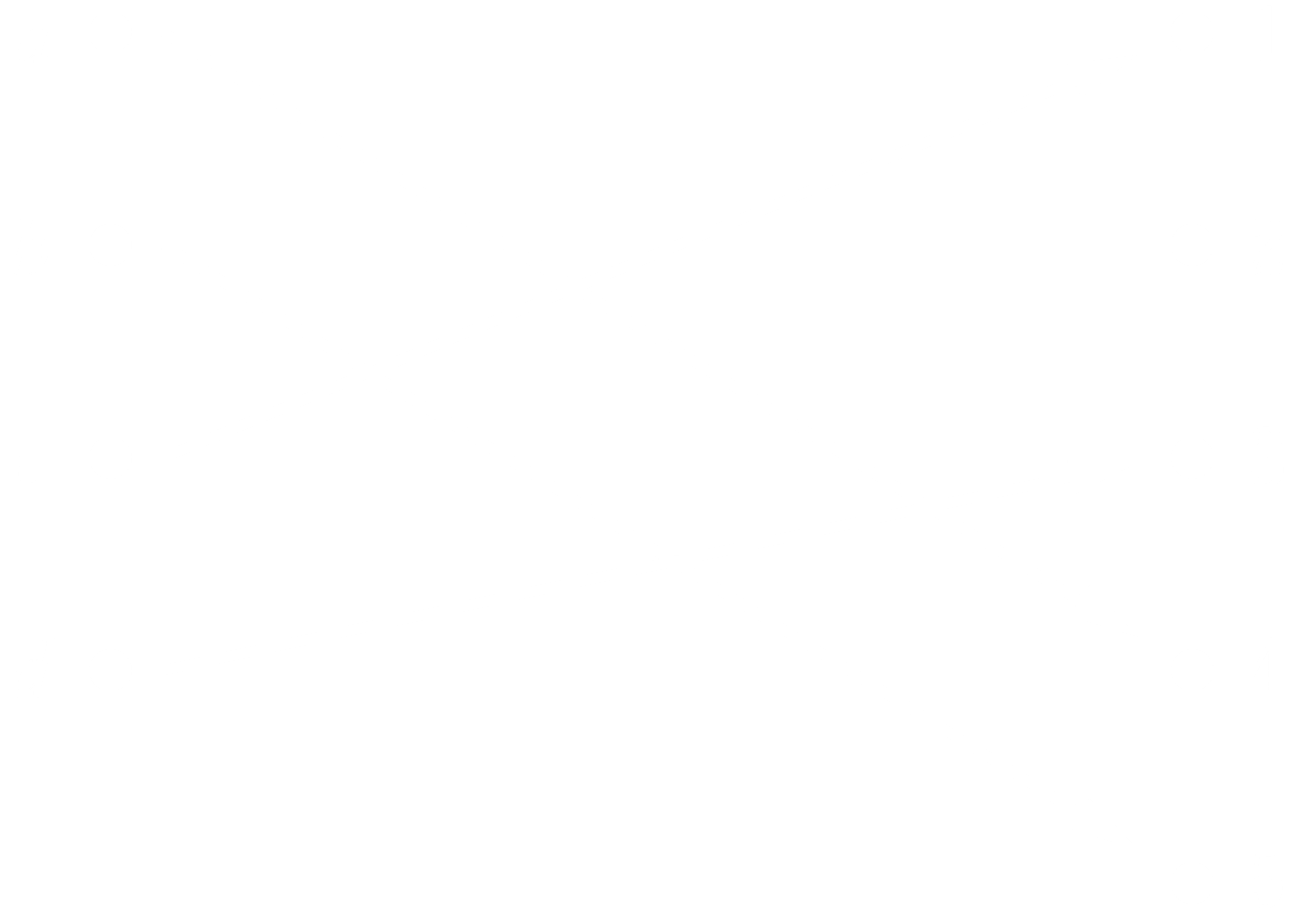
- union

- intersection

## Chapter 2.3: Functions

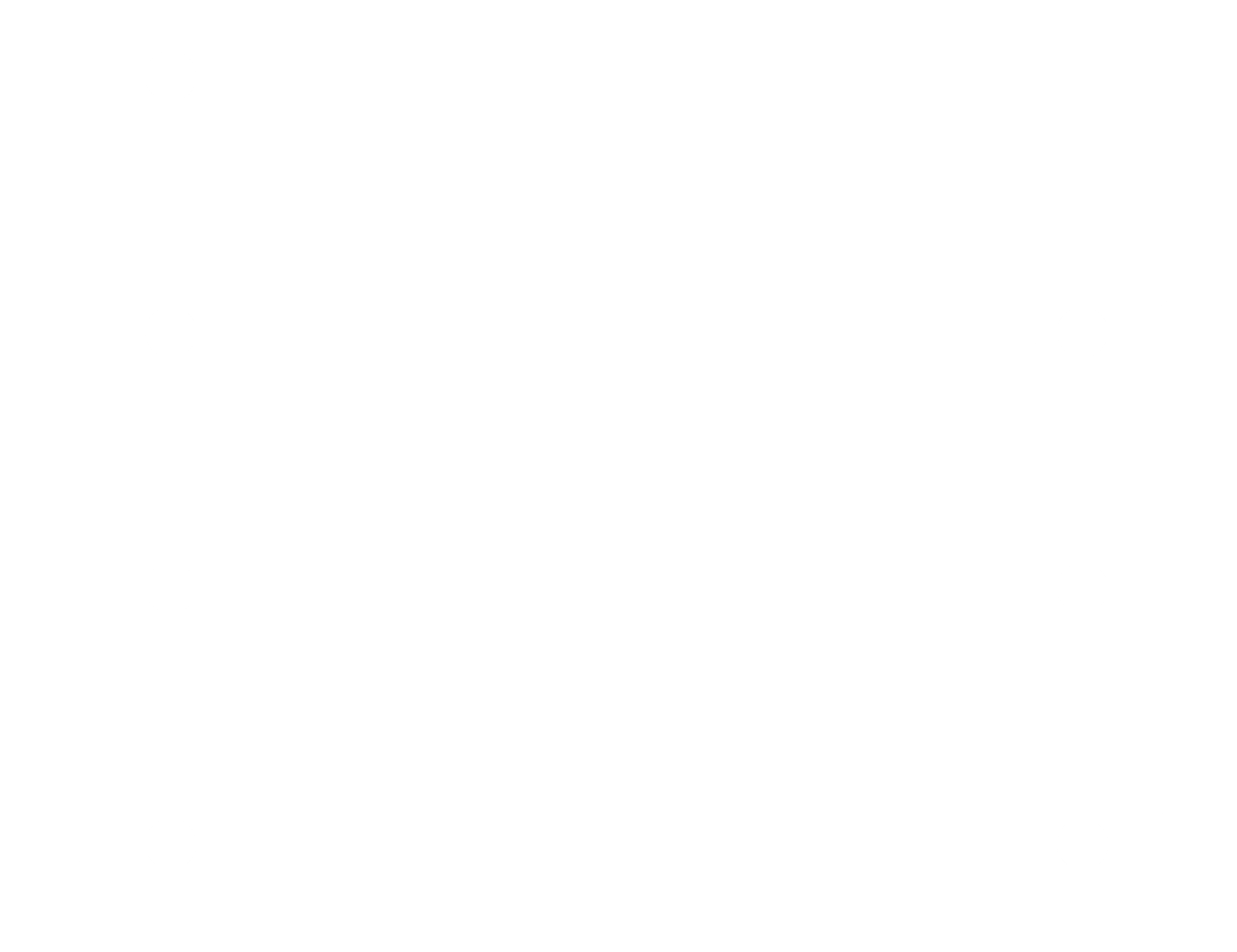
### One-to-One Functions or Injunctions

A function is said to be One-to-One if and only if implies that for all and in the domain of . Such a function is said to be injective. (Every value of is connected to exactly one result.)



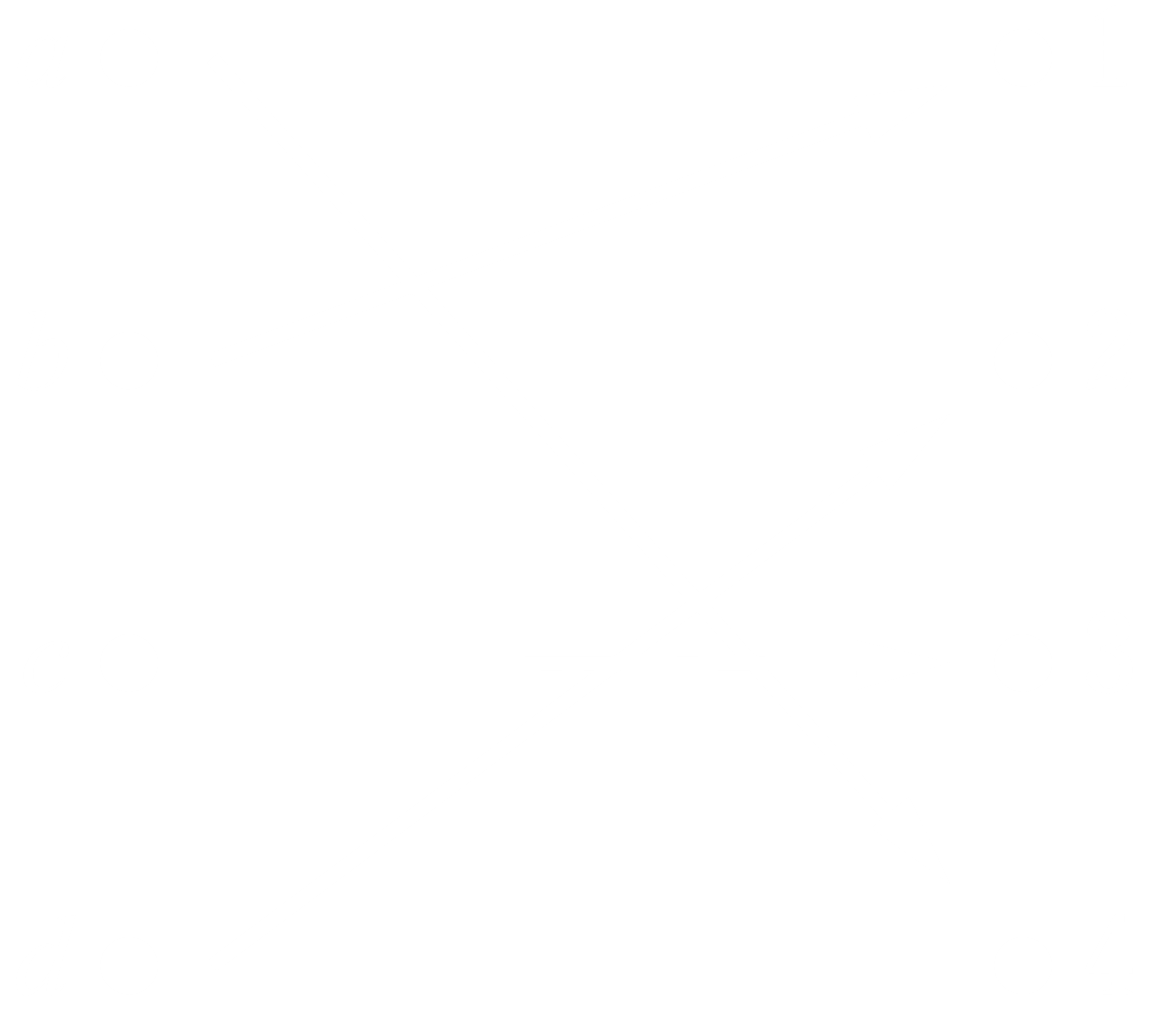
### Onto Functions or Surjective Functions

A function from to is said to be Onto if and only if for every element , there exists at least one element , such that . Such a function is called a surjection. (Every result is connected to at least one value of , but not necessarily just one.)



### One-to-One Correspondence or Bijections

A function is a One-to-One Correspondence if it is both One-to-One and Onto. Such a function is said to be bijective. (Every value of is connected to a result and every result is connected to a value of .)



### Inversion

For a One-to-One Correspondence, every element of is the image of some unique element from . This allows us to reverse the correspondence.

For a function that is a One-to-One correspondence from the set to the set , the inverse function of is the function that assigns to an element from , the unique element in for which . The inverse function of is denoted by . Thus, .

### Flooring and Ceiling

The floor function assigns to the real number , the largest integer that is less than or equal to . Thus, .

The ceil function assigns to the real number , the smallest integer that is greater than or equal to . Thus, .

Proof:

If , show that

Let , where and . There are thus two cases to consider, either or .

For

(since will always be less than )

For

(since will always be less than )

Thus, for both cases,

### Composite Functions

For two functions and , the composite function is .

The composite function can also be written as .